Concerning the Ratio H_{c3}/H_{c2} in Superconducting Nb and Nb-O Solutions*

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The Saint-James–de Gennes relationship between H_{c2} and H_{c3} in type II superconductors has been verified in Nb and Nb-O interstitial solid solutions. The observed ratio $H_{c3}/H_{c2}=1.7_0$ is found to be independent of κ, the Ginsburg-Landau parameter, for niobium-containing oxygen in concentrations below the solubility limit at the temperature of introduction. Critical-current data I_c versus H above H_{c2}, analyzed in terms of a dimension of the superconducting sheath, (e.g., $J_s=I_c$ per cm width), reveal sharp breaks at H_{c3} when the material becomes normal. For a cold-worked specimen, two portions of the $J_s(H)$ curve may be identified, meeting at a definite field corresponding to H_{c3} calculated from theory. The upper portion of the curve first meets the normal state curve at a field H_n. The ratio H_n/H_{c2} is larger for cold-worked niobium than for cold-worked niobium-oxygen alloys.

INTRODUCTION

RECENTLY, Saint-James and de Gennes[1] have shown that in type II superconductors in parallel fields, nucleation of a superconducting sheath will occur at the field $H_{c3}=1.695H_{c2}$, where $H_{c2}=\kappa\sqrt{2}H_c$.[2,3] H_c is the thermodynamic critical field and κ, the Ginsburg-Landau parameter. For the surface perpendicular to the field, the sheath of superconductivity should not exist beyond H_{c2}, Abrikosov's upper critical field. These predictions have been well substantiated by experiment, primarily in superconducting, substitutional solid solutions.[4–8]

In interstitial solid solutions where sample homogeneity is more difficult to achieve, it is possible for the specimen surface to be characterized by one κ and the interior by another, artificially forcing the apparent ratio H_{c3}/H_{c2} in either direction.[9] The interstitial solutions are essentially supersaturated and because the interstitial atom has great mobility, it can easily segregate to structural irregularities such as dislocations.

In this report we show that the ratio H_{c3}/H_{c2} observed in pure niobium and niobium containing oxygen in a concentration below the solubility limit at the temperature of introduction is in good agreement with the theoretical value proposed by Saint-James and de Gennes and found to be independent of concentration and/or κ. The influence of cold work, quenching, and strain-aging on the ratio and on H_n, a field larger than H_{c3}, is also cited.

EXPERIMENTAL

The experimental details have been described elsewhere.[9] The specimens were rectangular in shape, made by pressing niobium[10] between highly polished stainless steel blocks. The ribbons were outgassed and annealed at a temperature 1800–2000°C for 24 h in vacuum less than 1×10^{-7} mm Hg. They were $7.1_1\times10^{-2}$ cm wide by $7.6_2\times10^{-3}$ cm thick. A ribbon specimen of $Nb_{0.9916}O_{0.0084}$ was prepared by adding oxygen at about 1100°C to a previously outgassed niobium ribbon; then homogenized at 1200°C for several hours. This sample was $1.1_4\times10^{-1}$ cm wide and $1.9_1\times10^{-2}$ cm thick.

Two field orientations were examined: (1) field parallel to the wide side of the sample ($H||$w.s.), and (2) field perpendicular to the wide side ($H\perp$w.s.). In either case, H was perpendicular to the direction of current flow. The two field orientations were made precisely reproducible by mounting the ribbon in a form permanently fixed between the pole pieces of a small Nb_3Zr wire-wound, iron-core magnet.

RESULTS AND DISCUSSION

Measurements of longitudinal voltages in the direction of J as a function of applied field for a given current and for field orientation (a) $H||$w.s. and (b) $H\perp$w.s. are illustrated in Fig. 1 for $Nb_{0.9916}O_{0.0084}$. In both cases, the sample has two opposite surfaces parallel to the field, although the demagnetization coefficient is less favorable when $H\perp$w.s. and the total surface parallel to the field is smaller. For the two field directions studied, H_{c3} is evaluated when the potential first attains the normal-state value. This method of determining H_{c3} has some uncertainty since the normal voltage is approached asymptotically.

An alternative measurement is the determination of critical current I_c as a function of field, choosing the critical current as that current which gives an arbitrarily

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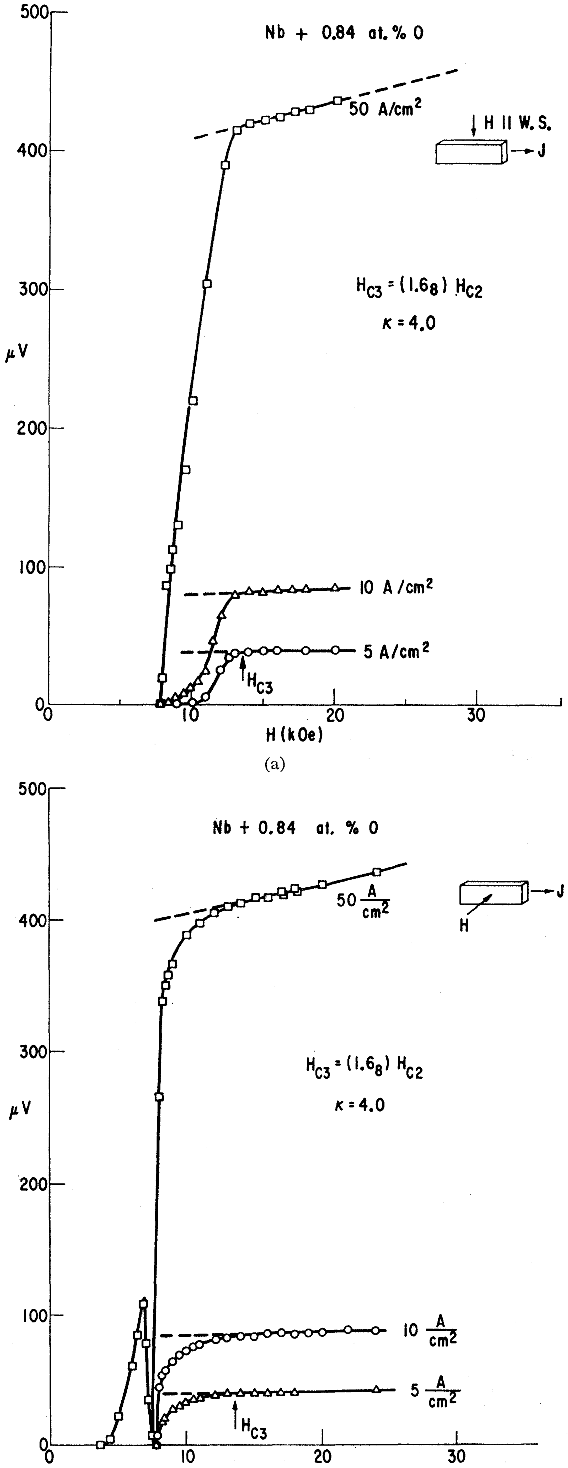

FIG. 1. The longitudinal voltage, along direction of J, observed as a function of current for a rectangular specimen $Nb_{0.9916}O_{0.0084}$ ($1.1_4\times10^{-1}$ cm wide$\times1.9_1\times10^{-2}$ cm thick). Distance between potential leads, 7×10^{-1} cm. (a) Field parallel to the wide side ($H\|$w.s.). (b) Field perpendicular to the wide side ($H\perp$w.s.). $H_{c3}\simeq13.0$ kOe, $H_{c2}=7.7_4$ kOe (magnetization), $T=4.2$°K.

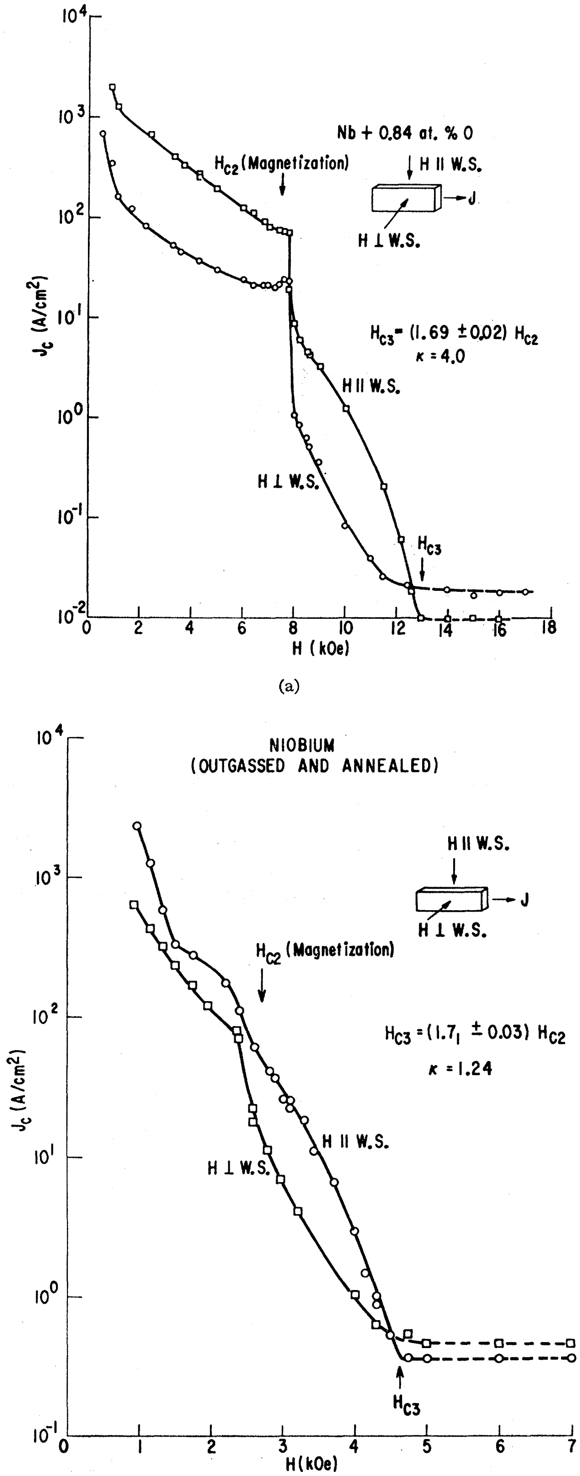

FIG. 2. The critical current J_c based on the cross-sectional area of the sample. (a) Specimen $Nb_{0.9916}O_{0.0084}$ ($1.1_4\times10^{-1}$ cm wide $\times1.9_1\times10^{-2}$ cm thick). (b) Nb (7.1×10^{-2} cm wide$\times7.6_2\times10^{-3}$ cm thick). $H_{c3}=4.6_2$ kOe, $H_{c2}=2.7_0$ kOe (magnetization), $T=4.2$°K. For fields above H_{c3} and for $H\|$w.s. the broken horizontal straight line represents normal state behavior.

chosen voltage. An arbitrarily chosen voltage is the minimum detectable voltage above noise level. For $H||$w.s., this voltage decision level was found to be minimum, equal to approximately 0.01 μV. For cold-worked ribbons, the minimum voltage above noise level was consistently higher, attaining maximum values of 0.1 μV when $H\perp$w.s. These variations in noise level, although not understood, are too small to alter the conclusions following below. Some typical I_c data in terms of critical current density, J_c (defined as the critical current per cross sectional area of the sample) as a function of field, are summarized in Fig. 2(a) and 2(b) for $Nb_{0.9916}O_{0.0084}$ and pure Nb, respectively. Above H_{c2}, J_c is seen to fall with increasing H until a value independent of field is reached. For $H||$w.s. this value of current density corresponds to that current in the normal state giving the arbitrary decision level. In this region, the sample is not superconducting. For $H||$w.s., the J_c versus H curve approaches the normal state curve rather decisively permitting a more accurate evaluation of H_{c3}. The ratio, H_{c3}/H_{c2}, evaluated from these data, is independent of κ in these two samples and in good agreement with the Saint-James and de Gennes theoretical value. H_{c2} has been determined from magnetization data.

If a superconducting sheath exists in the region $H_{c2}\leq H\leq H_{c3}$, as predicted, it would be more appropriate to analyze I_c data in terms of some dimension of this sheath pertaining only to the surfaces parallel to the applied field for a given orientation. A simple analysis would be the evaluation of the critical current flow associated per unit width of sheath, or $J_s=I_c$/cm width. J_s evaluated in this manner is summarized in Fig. 3 as a function of H for $Nb_{0.9916}O_{0.0084}$ and for $H||$w.s. and $H\perp$w.s. For $H||$w.s., the curve above H_{c2}, approximately linear on the semilog plot, forms an abrupt break with the horizontal normal-state curve. The field at which this break occurs corresponds to H_{c3}, calculated. Within experimental error, the approximate agreement in $J_s(H)$ for the two field orientations between H_{c2} and H_{c3} is evident. Similar results have also been obtained for pure niobium. This behavior indicates that the surfaces perpendicular to the applied field are, apparently, not contributing any remanent superconductivity above H_{c2} in agreement with theory.[1]

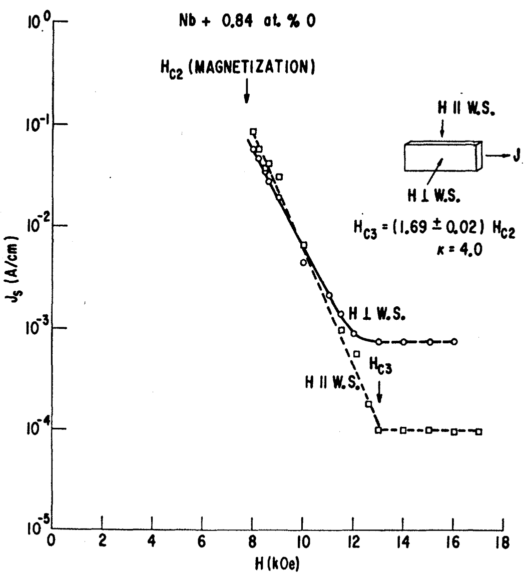

FIG. 3. The critical current J_s calculated from I_c per unit width of superconducting sheath. Thickness of sheath assumed independent of field and direction for $Nb_{0.9916}O_{0.0084}$ ($1.4_1\times10^{-1}$ cm wide$\times0.9_1\times10^{-2}$ cm thick). $T=4.2°$K. Horizontal curve represents normal-state behavior.

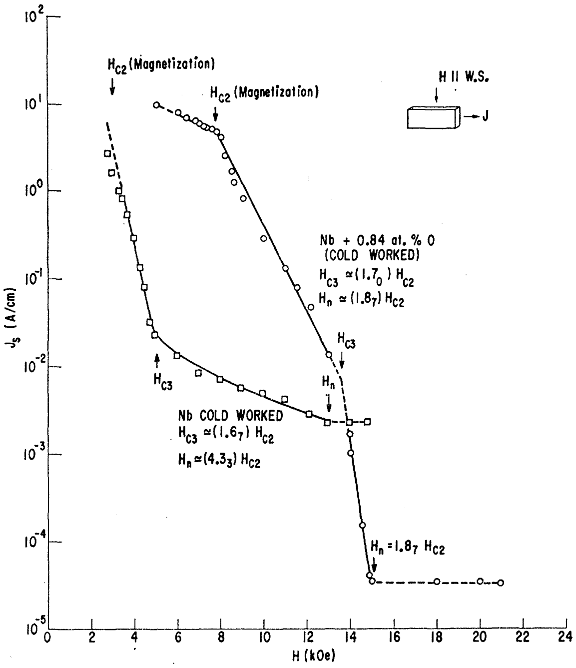

FIG. 4. The critical current J_s calculated from I_c per unit width of superconducting sheath for (a) cold-worked Nb ($1.3_7\times10^{-1}$ cm wide$\times8.8_9\times10^{-3}$ cm thick), (b) cold-worked $Nb_{0.9916}O_{0.0084}$ ribbons ($1.1_9\times10^{-1}$ cm$\times1.5_2\times10^{-2}$ cm); $T=4.2°$K.

The results summarized in Fig. 4 were obtained on Nb and $Nb_{0.9916}O_{0.0084}$ ribbons cold worked by compression and again evaluating J_s for $H||$w.s. Above H_{c2} and fields below the normal-state behavior, two distinct portions of the curve may now be identified. The field at the junction of these two segments corresponds to H_{c3} calculated from the Saint-James and de Gennes theory. The slope of the linear curve, dJ_s/dH, for either sample is approximately the same as that obtained from the data prior to the deformation. It is larger when κ is smaller. The absolute value of J_s, however, appears to be larger after being cold worked.

The upper portion of the curve above H_{c3} terminates at the normal state at a field H_n. H_n is larger relative to H_{c2} for the cold-worked niobium than for cold-worked $Nb_{0.9916}O_{0.0084}$ solid solution.

Deformation produces inhomogeneous distribution of dislocations; therefore, an inhomogeneous mean free path and an inhomogeneous κ. This can cause H_n/H_{c2} ratios greater than the ideal value 1.69. However, in an alloy, the mean free path is controlled primarily by solute concentration and variations in dislocation density cause much smaller variations in κ than in pure material. Therefore, in the Nb-O solution one would expect the ratio H_n/H_{c3} to be smaller than that observed in Nb.[11]

In an experiment on varying the degree of segregation at dislocations by (a) quenching, (b) cold working, and (c) strain aging (heating 3 h, 170°C), marked changes have been observed in the resistivity in the mixed state between H_{c1} and H_{c2} (see Figs. 16 and 17, Ref. 9) for $Nb_{0.993}O_{0.007}$. After quenching, $H_{c3}/H_{c2}=1.7_1$. Cold working the quenched sample increased H_n to approximately 1.8_3 H_{c2}, while strain aging may increase H_n slightly ($H_n \simeq 1.8_8 H_{c2}$).

Analysis of I_c data above H_{c2} in terms of J_s may help to elucidate some of the higher values reported for H_{c3} in type II superconductors (e.g., see Ref. 7).

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Perturbation Approach to the Diffraction of Electromagnetic Waves by Arbitrarily Shaped Dielectric Obstacles*

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A perturbation method is developed to consider the problem of the diffraction of electromagnetic waves by an arbitrarily shaped dielectric obstacle whose boundary may be expressed in the general form, in spherical coordinates, $r_p=r_0[1+\delta f_1(\theta,\phi)+\delta^2 f_2(\theta,\phi)+\cdots]$ where r_0 is the radius of an unperturbed sphere and $f_n(\theta,\phi)$ are arbitrary, single-valued and analytic functions. δ is chosen such that

$$\sum_{n=1}^{\infty} |\delta^n f_n(\theta,\phi)| < 1,\ 0\leq\theta\leq\pi,\ 0\leq\phi\leq 2\pi.$$

Detailed analysis is carried out to the first order in δ. Procedures to obtain higher order terms are also indicated. The perturbation solutions are valid for the near zone region of the obstacle as well as for the far zone region and they are applicable for all frequencies. Possible applications of this perturbation technique to elementary-particle scattering problems and other electromagnetic scattering problems are noted.

I. INTRODUCTION

THE exact solution of the problem of the diffraction of electromagnetic waves by an obstacle of given shape and electromagnetic properties can be found only in a few cases.[1,2] For example, the diffraction of waves by a conducting or dielectric sphere, by dielectric coated spheres and by a perfectly conducting disk are the few three-dimensional problems that have been solved rigorously. The need for approximate methods to treat the more general cases of diffraction from arbitrarily sphaped obstacles is quite apparent. The variational principles[3,4] provide a very powerful tool in obtaining an approximate expression for the scattering cross section; but it is not possible to derive from the variational principles a description of the electromagnetic fields. Furthermore, the success of the variational approach depends to a great extent on the trial function. At low frequencies, the Rayleigh method is very useful.[5,6] However, the solutions of Laplace's equation are still required. At very high frequencies, the treatment of diffraction problems by geometric and physical optics techniques developed by Fock[7] and Keller[8] is very successful. An approximate or perturbation method in the medium frequency range still remains to be found.

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